Math Challengers Assign 11 Workshop on COUNTING and PROBABILITY

A) Counting Problems:

1. Montreal and Toronto are playing in the baseball World Series. (Games cannot end in a tie, and the first team to win four games wins the Series.) How many different outcomes are possible? Here are a couple: MMMM (Montreal won in four straight) and MMMTTTT (Montreal won the first three, but Toronto came back to win the Series).

2. A bridge fanatic has five friends, who come every day to the fanatic's house to play. It takes four people to play a hand of bridge. The fanatic takes part in every hand, but two of the friends have to sit out each hand. This month, the fanatic played in 1500 hands. If each of her/his friends played in equal number of hands, how many more hands did the fanatic play in than any of his/her friends?

3. A soccer ball is sewn together from 32 pieces. Twelve of these are regular pentagons, and the rest are regular hexagons. The pentagons and hexagons all have the same side length. A seam has to be sewn wherever two pieces meet. How many seams are there?

4. Three movie fans were talking about the movies they had seen this year at the local theatre. The first had seen 50, the second 60, and the third 70. Between them they saw every one of the 99 movies shown at the theatre this year. Call a movie very popular if it was seen by all three, and unpopular if it was seen by only one. How many more movies were unpopular than very popular?

5. How many times between noon and midnight are the hour hand and the minute hand of a clock at right angles to each other?(b) Between 1:00 and 2:00 when does the minute hand and the hour hand form a right angles?

6. A point in the plane with integer coordinates is called a lattice point. How many lattice points are there inside or on the circle that has equation $x^2 + y^2 = 10$?

7. (a)) How many triples (a,b,c) of positive integers are there with $a + b + c = 101$?
(b) and with $a < b < c$

8. How many possible yearly calendars are there?

9. Let N be the number whose decimal expansion has the shape 999---998 (221 9's followed by an 8). Find the sum of the decimal digits of N^2.

10. Someone wrote down, in decimal notation, all the numbers from 0 to 9999. What is the sum of all the digits that were used?

11. A rectangle is divided into 20 small rectangles by drawing 3 lines parallel to its base and 4 lines perpendicular to its base. What is the total number of rectangles available?

12. (a) How many solutions in positive integers are there for the equation $x + y + z = 1999$? (The solution $x = 3, y = 3, z = 1993$ is to be considered different from $x = 3, y = 1993, z = 3$.) (b) How many solutions are there in integers all ≥ 0?

B) Probability Problems:

1. In front of you there are two opaque containers. One of the containers (you don't know which) has 2 twenty-dollar bills and 5 five-dollar bills. The other container has 3 twenty-dollar bills and 11 five-dollar bills. You are given the following options: (a) Pick a container, reach in and pick a bill at random, which you get to keep or (b) Pour the contents of one container into the other, pick a bill at random, and keep it. Which option is better? How much better?

2. We select a point "at random" inside a given square. What is the probability that the chosen point will be closer to the center of the square than to any of the four corners?
(b) For a rectangle? (c) For equilateral triangle?

3. We own eight CDs, each with its own case. After a party, the CDs were put into cases at random. (a) What is the probability that all of the CDs but one are in the right cases? (b) What is the probability that all but two are in the right cases? (c) What is the probability that all but three are?

4. With a pair of ordinary dice, the probability that a sum of 3 is tossed is 2/36, the probability that a sum of 8 is tossed is 5/36, and so on. For a certain game, you need sums 1, 2, … , 11, 12 to be equally likely. How can you relabel the faces of the dice so as to obtain this?

5. A and B are playing a game in which a fair coin is to be tossed repeatedly. A wins a point for each head tossed, and B wins a point for each tail tossed. Whoever first gets 6 points wins the game. (a) After six tosses, A is leading 4 to 2. What is the probability that B comes back to win the game? (b) When A and B started the game, each agreed to pay the other 100 gold coins if the other won. Unfortunately, when A was leading 4 to 2, B dropped dead. How much can A legitimately claim from B's estate?

6. Assume that in a basketball game the better team always wins. Eight teams reach the last phase of a high school tournament. After that, any team losing a game is eliminated. So in the quarterfinals four teams will be eliminated, and in the semifinals two more. In the final a champion and runner-up (the team that played in the final, but lost) are declared. What is the probability that the runner-up team is in fact the second-bets team?

7. There are 52 cards in a standard deck, among them 12 face cards (Jacks, Queens, and Kings). What is the probability that among the first three cards dealt from a well shuffled deck, there is at least one face card?

8. A bowl contains 49 tickets, numbered from 1 to 49. You remove two tickets chosen at random. What is the probability that the numbers you get are within 10 of each other?

9. X rolls one fair die, then Y rolls two. Y wins if at least one of her dice beats X's, and otherwise X wins. What is the probability that X wins?

10. Dice A, B, C are cubes, as usual, but with not quite the usual face markings. Die A has two 2's, two 4's, two 9's. die B has two 3's, two 5's, two 7's. Die C has two 1's, two 6's, two 8's.

Players X and Y play the following game. Player X selects one of the three dice. Then, knowing X's choice, Player y selects whichever of the remaining dice she wants. They each toss their die once. Whoever comes up with the bigger number wins. Show that whatever the Player X selects, Player Y can choose a die that has a better than 50% chance of beating Player X.